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BER Analysis of OFDM with Raised Cosine Window and Equalization using Rician Channel

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Abstract

Wireless communications is emerging as the largest sectors of telecommunications industry. It is the one area that promises a lot of prospects for growth in the coming century. The major challenges for the future research and study in this field include utilizing the limited available spectrum efficiently and improved link reliability between communicating equipment. The environment in which the communication equipments are required to be operating suffers from fading and interference from other users. Frequency selective fading channels can model such hostile conditions that exist in the environment. The Orthogonal Frequency Division Multiplexing (OFDM) technique that is a multi-carrier technique for data transmission is best suited for such channel models. OFDM is thus characterized by the presence of many sub-carriers. These sub-carriers among the different users bear an orthogonal relation with each other. Here the user spectrums have the flexibility to overlap thereby leading to an enhanced spectrum efficiency of the system. This flexibility is provided by this orthogonal relationship between the sub-carriers as it is necessary in overcoming deteriorating effects such as ICI or Inter-Carrier Interference which is responsible for reducing the BER (Bit Error Rate) performance of the system. Here, it is observed that for the traditional OFDM system the SNR of 55 dB is achieved as compared to the simulated result (involving RC (Raised Cosine) window and Equalization techniques) of an SNR value of 31 dB; for the bit error rate value of $10^{-2.9}$ under Rician channel. So, an improvement of 24 dB is achieved with the proposed OFDM system. Thus, the BER performance of the system shows improvement.

Keywords: OFDM, Rician, Equalization, RC Windowing.

Introduction

OFDM (Orthogonal Frequency Division Multiplexing) which is a modulation method based on multiple carriers provides us with a considerably high efficiency of spectral usage, tolerance against multipath delay spread, immunity against frequency selective fading channels and efficiency of power. Due to such benefits OFDM is being implemented in applications that require very high data rate communications, Digital Video Broadcasting (DVB) and mobile communications [1]. OFDM has been adopted in the next generation of wireless communications, 4G for its abilities to fight large and small scale fading with low implementation complexity of the system. OFDM has better spectrum utilization efficiency as compared to Frequency Division Multiplexing (FDM). This is the result of the orthogonal relationship that the multiple sub-carriers have with respect to each other thus avoiding adjacent channel interference. The performance of multi-carrier communication systems such as OFDM is calculated by finding the relationship between BER (bit error rate) and the SNR (signal-to-noise ratio). Accurate simulation of the considered channels is very important in the evaluation of the system performance and the design process for the communications equipment and other related systems. Fading or loss of signals is a very important phenomenon that leads the design engineers to define fading models that try and describe the patterns of fading in different types of environments. Though no model can precisely describe an environment, still they strive to achieve as much precision as possible. The better the ability of a channel model to describe a fading environment, the better the said channel is

compensated with other signals so that on the receiving side the signal is as close to being error free as possible. This would mean better clarity of the received signal with high accuracy of the transmitted data or lower Bit Error Rate (BER).

Multipath fading is a cause for major concern for the design process of communicating equipment. It causes severe noise at the receiver thus undermining the integrity of the communication system. The Rician fading channel is observed in a system with Line of sight (LOS) propagation and scattering. The Rician fading channel correctly models the environment that our communication equipment has to operate in. The Rician fading channel model is quite similar to Rayleigh model, with one exception that the Rician model is characterized with the presence of a dominant component which has a strong presence in the received signal. This component which is the dominant part of the received signal is the LoS or the line of sight component [2].

In this paper, a Rician fading channel model is considered while calculating the BER versus SNR results for OFDM system using 16QAM modulation scheme. A windowing technique namely the Raised Cosine (RC) windowing technique is applied apart from equalization operation at the receiver. High frequency spectral noise is generated due to transitions in the time waveform that are very fast. This operation is done to mitigate the effects of ISI to decrease the probability of error that would occur otherwise. Zero forcing (ZF) equalization is also utilized in conjunction with the windowing technique.

OFDM System Fundamentals
OFDM
OFDM (Orthogonal Frequency Division Multiplexing) is a method to modulate and multiplex simultaneously. OFDM operates on the orthogonalization of the sub-channels. For lower complexity in achieving the orthogonality property, Fast Fourier Transform (FFT) is preferred. As the high data rates are transmitted in serial form, they are converted into many parallel lower data rates using different modulation techniques for each sub-channel. The signals that are generated may be taken from different sources. These signals which ultimately form a sub-sets for one composite signal are multiplexed together. These signals that split-up into channels are then modulated and then multiplexing is performed to achieve the final signal that is transmitted. However, a separation still exists in the frequency domain. In order for the

maintenance of exclusivity a sufficient separation is maintained between these sub-carriers. The guard intervals are also provided which also lead to the poor spectral efficiency. But the orthogonality principle is used in order to boost the spectrum utilization. The orthogonality principle is very important for the success of the OFDM system performance.

Figure 1:
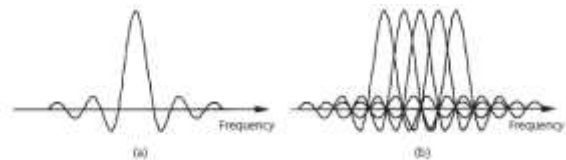
OFDM Spectrum showing Orthogonal Sub-Carriers

The carriers bear an orthogonal relationship to one another. Due to this property, for the channel that is frequency selective various issues such as the Inter Symbol Interference are reduced. The rate of transmission increases due to the minimal spectral requirements; which is the result of the overlapped sub-channels. The OFDM system block diagram is shown below.

Figure 2:
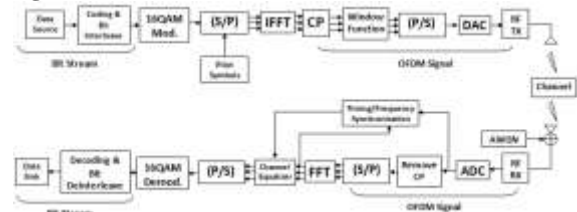
OFDM System Block Diagram

Several methods of coding or encrypting the message are also employed that further enhance the system robustness. Another issue that is commonly encountered is the ICI (Inter Carrier Interference). This problem arises due to the sensitivity that the system bears to the carrier frequency offset. The orthogonal relation between the various sub-carriers leads to a reduction in this problem as well.

Mathematical Analysis
Mathematically, each carrier in the OFDM composite signal might be approached as a wave in complex form.

$$S_c(t) = A_c(t)e^{[j\omega_c(t)+ \varphi_c(t)]} \tag{1}$$

The real signal forms the real part. $\varphi_c(t)$ & $A_c(t)$

represent the phase and amplitude of the carrier which generally are variable depending on the symbol. The values of the parameters remain uniform for the entire symbol period (t) duration. Due to the presence of several different carriers, the composite OFDM signal becomes,

$$S_s(t) = \frac{1}{N} A_N \, e^{[j\omega_n(t)+\,\varphi_n(t)]} \qquad (2)$$

Where, $\omega_n = \omega_0 + n\Delta\omega$.
This signal is continuous. Considering each component for the duration of a single symbol, the amplitude and phase variables take up uniform values which depend on the particular carrier frequency.

Now, sampling the signal using sampling frequency of $1/_T$, the resulting signal becomes,

$$S_s(kt) = \sum_{n=0}^{N-1} A_N e^{[j(\omega_0+n\Delta\omega)kT+\,\varphi_n(t)]} \qquad (3)$$

Now, it being very easy for sampling over one data symbol period hence, the time over which the signal is analyzed is restricted to N samples. Therefore, we have,

$$t = N.T \qquad (4)$$

$$S_s(kt) = \sum_{n=0}^{N-1} A_N e^{[j(n\Delta\omega)kT+\,\varphi_n(t)]} \qquad (5)$$

Equation (5) is considered for comparison with the Inverse Fourier Transform general form equation.

$$g(kT) = \frac{1}{N} \sum_{n=0}^{N-1} G\left(\frac{n}{NT}\right) e^{\frac{2\pi}{N}kn} \qquad (6)$$

In equation (6), the function $(A_N e^{j\varphi_n})$ is more of a frequency domain signal definition and the time domain definition being, $S(kT)$.

Equations (5) and (6) bear some equivalence if the following condition is met,

$$\Delta f = \frac{\Delta\omega}{2\pi} = \frac{1}{NT} = \frac{1}{\tau} \qquad (7)$$

This condition is the same as the orthogonality condition [3]. Thus, one advantage of the orthogonal relationship between the sub-carriers is that the composite OFDM signal could be defined using the Fourier Transform procedures.

Fading and Multipath

Fading is defined as the signal distortion experienced while traversing through various media or channels. In communications, the multipath propagation is the prime reason for the fading phenomenon. In order to understand fading, we must understand the concept of multipath propagation. Multipath is that phenomenon that entails the radio signal as is sent from the transmitter, reaches its destination of the receiver traversing two or more different ways or paths. It is caused by reflection, both ionospheric and terrestrial, refraction and atmospheric ducting. Multipath causes phase shifting of the signal and interference that is both constructive and destructive. The signal distortion that results due to the multipath effects is known as fading.

Additional paths for propagation of radio waves that are beyond the line of sight also arise due to various phenomenon such as Diffraction, reflection and scattering.

Fading Channel Models

The amplitude and the phase of the signal fluctuates due to multipath fading as a consequence of the vector addition of several signals that arrive bearing different phases at the receiving station. This phase difference is due to the different paths that are taken by the signal. As the phases change rapidly, the amplitude of the signal that is received at the receiver varies rapidly. These variations are modeled with a particular distribution as a random variable. Rayleigh distribution is the most common for multipath fading. The Rayleigh Fading probability density function (PDF) is [4],

$$f_{Rayleigh}(r) = \frac{r}{\sigma^2} exp\left(-\frac{r^2}{2\sigma^2}\right), r \geq 0 \qquad (8)$$

Here, we assume that similar attenuation afflicts all signals however they all have different phases when they arrive at the receiver. Corresponding to the amplitude of the signal, the random variable is 'r'. The quadrature and in-phase variance is 'σ^2'. The received signal has a complex envelope which is considered to be distributed uniformly in $[0,2\pi]$.

When a strong signal component that is also the line of sight (LOS) component exists, the distribution is Rician. The probability density function (PDF) of such function is,

$$f_{Rician}(r) = \frac{r}{\sigma^2} exp\left(\frac{-(r^2+A^2)}{2\sigma^2}\right) I_0\left(\frac{Ar}{\sigma^2}\right), r \geq 0, A \geq 0 \qquad (9)$$

Here, the quadrature and in-phase variance is 'σ^2'. The dominant path signal amplitude is given by 'A', and the Bessel function which is modified and of the zero-order is given by, 'I_0'. In general, the depth of the fading is predominantly reduced by the dominant path and a superior performance is achieved by the Rician Fading as compared to Rayleigh Fading in terms of the Bit Error Rate (BER). Here, the size of the cell determines the probability of having a strong line of sight (LOS) component. The smaller is the size of the cell, the higher is the probability of having LOS path. The Rician PDF is reduced to Rayleigh PDF in the absence of any dominant path. When the value of A is large compared to σ, the distribution is approximately Gaussian. Thus, since Rician distribution covers the Gaussian as well as Rayleigh distributions; mathematically the Rician fading channel can be considered to be the general case [5].

Rician Fading Channel
        A Rician Fading model is applied in simulating the environments that produce the multipath components of the signal and have a dominant component which is the LOS component amongst signals that are received at the receiver [6]. This dominant component is a stationary or a non-fading signal.

    Another name for the dominant line of sight component is the 'specular component' and for the multipath component is the 'scatter component' or the 'random component'. The mean of the amplitude distribution for the former component is non-zero and for the latter component is zero.

    Representation of the signal as is received through the Rician distribution is,

$$s_r = re^{j(\omega_0 t + \theta)} + ae^{j\omega_0 t} = \rho e^{j(\omega_0 t + \varphi)} \qquad (10)$$

    Where, a represents the amplitude of the signal in its attenuated form, where the random value for phase is 'θ'. The marginal value for the Probability Density Function would be $\left(f_p(\rho)\right)$, which is named Rician, with the random value of the amplitude ρ is [7],

$$f_p(\rho) = \frac{\rho}{\sigma^2} exp\left(-\frac{\rho^2 + a^2}{2\sigma^2}\right) I_0\left(\frac{a\rho}{\sigma^2}\right) \qquad (11)$$

    We define the main component power as, 'p_d', p as the instantaneous received power, the scattered components power as, '$\bar{p}_s \equiv \sigma^2$', the mean of the received power as '$\bar{p} = p_d + \bar{p}_s$' and the Rician factor is taken as K,

$$p_d = \frac{a^2}{2}, p = \frac{p^2}{2} \qquad (12)$$

$$K = \frac{p_d}{\bar{p}_s} = \frac{p_d}{\sigma^2}, \sigma^2 = \frac{\bar{p}}{1+K} \qquad (13)$$

    Applying the Jacobian to equation (11)

$$f_p(p|\bar{p}, K) = \frac{1}{\sigma^2} exp\left(-\frac{p}{\sigma^2}\right) e^{-K} I_0\left(\sqrt{\frac{4Kp}{\sigma^2}}\right) \qquad (14)$$

    Equation (14) gives the probability density function (PDF) of power and is also known as "non-central Chi-square pdf" [8].

Windowing
        The pulse shaping or windowing technique is used to reduce the sensitivity to linear distortion at the transmitter side. At the transmitter, after IFFT modulation, the OFDM signal is cyclically extended and then the applied window function shapes the CP part keeping the original part of the signal unchanged. The basic purpose of the windowing technique is to make the spectrum go down rapidly, such as the amplitude of the OFDM symbol goes smoothly to zero at the symbol boundaries. If the windowing technique is not used, then the spectrum that is out-of-band decreases rather

slowly due to a phase transition that is sharp at the symbol boundaries [9].

Conventional Windowing Techniques
        Consider an OFDM symbol with J available sub-carriers. 'J_{CP}' as the size of the cyclic prefix, 't_S' as the duration of the symbol, and we assume the guard carriers as 'J_G'. As is observed in traditional techniques for windowing, for the OFDM system between the two consecutive symbols the transition is smoothened for side-lobe suppression. An additional window time is thus placed between two adjacent symbols for smoother transitions. During this interval the sub-carriers of the past symbol fade. This transition which is smooth is facilitated by multiplying the window function with the modulated symbols. These are further extended with the post-fix and the pre-fix. For the orthogonality to be maintained, overlapping happens between 'J_W' samples from the ending part of the symbol that has past and the 'J_W' samples from inception of the next OFDM symbol. As the symbol duration for the windowed OFDM symbol is windowed is $(J + J_{CP} + J_W)$, the traditional windowing technique leads to a decrease in the efficiency of the spectrum that depends upon 'J_W'. Also, as the data and the cyclic prefix of the composite OFDM still remain, the issues like the Inter Symbol Interference which result from the delay spread caused due to multipath is avoided. Now, filtering technique can also be used as an alternative to OFDM windowing in order to tailor the spectrum roll-off but, it is not as windowing technique can be carefully controlled [10]. In the filtering technique rippling effects (that may lead to symbol distortion in the OFDM symbol which in turn results in reduced immunity for the delay spread) in the region of the symbol known as the roll-off region are produced. Efforts should be made to avoid them.

Raised Cosine Windowing
        Consider The Raised Cosine window is so known because its individual function has a raised value of the cosine function. Therefore, the nil mark is just touched by the negative peaks of the window. The RC window is also known as the Hanning window. The mathematical representation of this window is,

$$w(t) = \begin{cases} 0.5 + 0.5\cos\left(\pi + \pi t /_{(\beta T_r)}\right), & 0 \le t \le \beta T_r \\ 1.0, & \beta T_s \le t \le T_r \\ 0.5 + 0.5\cos\left((t - T_r)\pi /_{(\beta T_r)}\right), & T_s \le t \le (1+\beta)T_r \end{cases} \qquad (15)$$

    Here, the symbol interval is represented by, 'T_r'. Now, its value is so chosen that it is of shorter duration as compared to the symbol duration for

OFDM. The roll-off region of the RC Window allows the possibility of partial overlap. The windowed OFDM symbol could be represented as,

$$R(t) = 2\,Re\left\{w(t)\sum_{n=0}^{N-1} d_n e^{j2\pi nt/T}\right\}, for\ 0 \leq t \leq T \quad (16)$$

Equalization

Equalization is a process of adding a digital filter at the receiving section that would approximate the inverse of the channel frequency response. Equalization is utilized to reduce the deteriorating issue of ISI (Inter Symbol Interference) to decrease the probability of errors that occurs otherwise. The primary cause of ISI is multipath fading that distorts the transmitted composite Orthogonal Frequency Division Multiplexing (OFDM) signal which also results in the production of bit errors at the receiving section [11]. Thus, in order to minimize this error or disparity between the desired output and the actual output, the technique of channel equalization is used. The filter coefficients are constantly updated in the equalization process. It must be kept in mind that this reduction of ISI has to be balanced with the prevention of an increase in the noise power.

Equalization is a filtering approach that can be accomplished both in the frequency and the time domain. The frequency domain equalization is preferred over that in the time domain due to its simplicity in application. The equalization process is categorized as liner or non-linear. This distinction depends upon the use of the output of the equalizer. Another filter design concept would be the number of taps. The number of taps in the equalizer is limited by the maximum amount of delay that the channel offers. Now, the delay intervals that are equalized have a value that is less than or equal to the maximum amount of delay that the structure of the filter presents. One of the most useful equalization algorithms is the Zero Forcing (ZF) equalization.

Zero Forcing Equalizer

Consider In the ZF (Zero Forcing) Equalization process, the coefficients [12] are so chosen that the combined channel samples are forced to zero, along with the impulse response of the equalizer. These zero values are obtained at all but one of the sampled points in the delay line that is tapped. With the equalization process the channel response satisfies the first criterion of Nyquist i.e.,

$$H_{CH}(f)H_{EQ}(f) = 1 \quad (17)$$

The zero forcing equalizer that is designed using the above equation is not capable of removing all ISI due to the finite length of the filter. This method has some demerits that include large inversion of gain for

some of the sub-carriers when their value is low which leads to amplification of noise in the sub-carriers. Thus, in this work the concept of windowing is used in conjunction with the equalization process so as to make possible betterment in the values of BER for OFDM communication system.

Simulation and Results
BER for OFDM using Rician Fading Channel

Considering M-ary QAM mapping scheme in Rician fading channel, the analytical expression for bit error rate is given by the following expression,

$$BER_{Rician} = \frac{1}{2}erfc\left[\sqrt{\frac{K\left(\frac{E_b}{N_0}\right)}{\left(K+\frac{E_b}{N_0}\right)}}\right] \quad (18)$$

Where, K - is the Rician factor. It is defined as the ratio of the LOS signal component power to the random signal component power.

$$K = \frac{m^2}{2\sigma^2} \quad (19)$$

Here, 'm' represents the non-zero mean and 'σ^2' is the variance.

Results and Discussions

For simulating the results, 16-QAM mapped OFDM system was considered along with RC (Raised Cosine) Windowing and the Zero Forcing Equalization technique using the Rician Fading channel. Figure 3 depicts the BER value for the traditional OFDM which is mapped using 16-QAM scheme and using Rician Channel.

Figure 3:
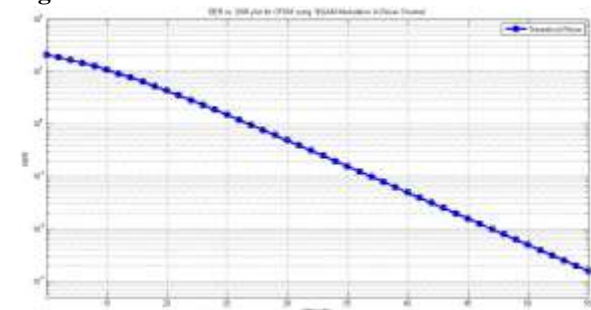
BER versus SNR for OFDM with 16-QAM using Rician Fading channel

In the figure 4, the bit error rate (BER) versus signal-to-noise ratio (SNR) for OFDM using RC window function and zero-forcing equalization in Rician fading channel model is shown. This result is compared with the bit error rate result for traditional OFDM. For the traditional OFDM it is observed that SNR of 55 dB is achieved as compared to the simulated result (involving RC (Raised Cosine)

window and Equalization techniques) of an SNR value of 31 dB for the bit error rate value of $10^{-2.9}$ under Rician channel. So, an improvement of 24 dB is achieved with the proposed OFDM system.

Figure 4:
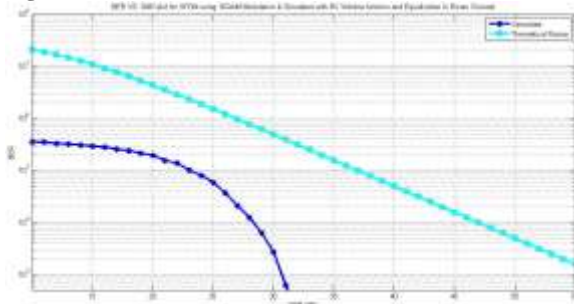
BER vs. SNR OFDM using 16-QAM with RC windowing and equalization along with traditional OFDM system using Rician Fading channel

Conclusion

We have shown the bit error rate versus signal-to-noise ratio performance for the OFDM system of communication for Rician Fading Channel. The BER results for the traditional OFDM method was compared with those of the OFDM method implemented in conjunction with RC (raised cosine) windowing method and ZF (zero forcing) Equalization technique. By adding Raised Cosine (RC) Window along with Equalization function in OFDM system that is implemented using 16-QAM modulation at the BER value of $10^{-2.9}$, an improvement of 24 dB is achieved in the SNR value as compared to traditional OFDM system for Rician Fading Channel.

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